

Five-branes in M(atrix) Theory

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We propose a construction of five-branes which fill both light-cone dimensions in Banks, Fischler, Shenker and Susskind's matrix model of M theory. We argue that they have the correct long-range fields and spectrum of excitations. We prove Dirac charge quantization with the membrane by showing that the five-brane induces a Berry phase in the membrane world-volume theory, with a familiar magnetic monopole form.

1. Introduction

Recently Banks et. al. have proposed a definition of eleven-dimensional M theory in the infinite momentum frame [1], as a large N limit of maximally supersymmetric matrix quantum mechanics. This system has a rather unusual history – it was first studied as a regulated supermembrane theory [2], and later arose as the theory governing the short distance dynamics of D0-branes in type Ila superstring theory [3]. The results following from both studies fit naturally into their picture.

In this note we propose a definition of certain five-branes in this theory, and check a number of the known properties of the five-brane in M theory – in particular, both the particles in the supergravity multiplet and the supermembrane as defined in [1] see the correct long-distance metric and quantized magnetic four-form field strength. This also confirms that the supermembrane couples to the three-form gauge potential, which is not manifest in their definition.

The five-branes we discuss are those whose world-volume includes both light cone coordinates, or “longitudinal five-branes.” These are objects with zero longitudinal momentum in the ground state and thus should be considered as non-trivial backgrounds in the IMF. On general grounds, such a background should correspond to a modified Lagrangian.

The specific modification we propose is inspired by the analogous system in the type Ila string, interacting Dirichlet 4-branes and 0-branes. The 4-brane arises as an M theory five-brane wrapped around the eleventh dimension, and thus it has Kaluza-Klein excitations. These reduce to Ila 0–4 bound states, and one was exhibited in [4]. The short distance interaction between these objects and thus the existence of the bound state is entirely due to stretched open strings between the 0-branes and 4-branes. The lightest such strings form a hypermultiplet in the vector representation of the zero-brane gauge symmetry group.

Keeping only these lightest modes as additional M theory degrees of freedom leads to a theory with (generically) no new massless degrees of freedom, but a modified dynamics for the zero-branes and new bound states. The discussion of the five-brane Kaluza-Klein modes is very analogous to the discussion of supergravity KK modes in [1], and the arguments they give for re-interpreting zero-brane bound state dynamics as the IMF dynamics of particles in the supergravity multiplet apply here, allowing us to re-interpret the dynamics of the new zero-brane bound states as the IMF dynamics of particles on the five-brane world-volume, forming a tensor multiplet.

Thus, the addition of a five-brane to the background will be implemented by adding a hypermultiplet in the vector representation to the Lagrangian. The location and orientation of the five-brane will be encoded in the couplings and Lorentz properties of the

additional fields. By integrating out this hypermultiplet, we will derive both the world-volume spectrum and the long-range fields, as seen by particles and by membranes.

We propose the Lagrangian in section 2 and show that supersymmetry acts properly. In section 3 we consider the interaction with 0-branes, arguing both that the bound states exist and that the long range fields are correct. In section 4 we consider the membrane, and show that its global interaction with the five-brane magnetic field can be understood as a Berry phase for fermion zero modes on its world-volume. This leads to a very simple proof that it and the five-brane satisfy the Dirac condition with the minimal charge quantum. Section 5 contains conclusions.

2. Matrix-vector quantum mechanics

The model of [1] is a $U(N)$ gauged matrix supersymmetric quantum mechanics. The Lagrangian (eqn. 4.2 of [1]) is

$$L = tr \left[\frac{1}{2R} D_t X^i D_t X^i - \bar{\theta} \gamma_- D_t \theta - R \bar{\theta} \gamma_- \gamma_i [\theta, X^i] - \frac{1}{4} R [X^i, X^j]^2 \right] \quad (2.1)$$

with $D_t = \partial_t + iA$ and $\gamma_+ \theta = 0$. The SUSY transformation laws are

$$\begin{aligned} \delta_0 X^i &= -2\bar{\eta} \gamma^i \theta \\ \delta_0 \theta &= \frac{1}{2} \gamma_+ \left[D_t X^i \gamma_i + \gamma_- + \frac{1}{2} [X^i, X^j] \gamma_{ij} \right] \eta \\ \delta_0 A &= -2\bar{\eta} \theta \end{aligned} \quad (2.2)$$

(written δ_0 to distinguish them from the supersymmetries of the Lagrangian with the five-brane added). The 16 supersymmetries $\tilde{\eta}$ satisfying $\gamma_+ \tilde{\eta} = 0$ are realized trivially, while the others $\gamma_- \eta = 0$ anticommute to the Hamiltonian

$$H = R \, tr \left\{ \frac{\Pi_i \Pi_i}{2} + \frac{1}{4} [X^i, X^j]^2 + \bar{\theta} \gamma_- \gamma_i [\theta, X^i] \right\}. \quad (2.3)$$

In the eleven-dimensional interpretation, this is the light-cone Hamiltonian $P_+ = H$, while the longitudinal momentum $P_- = N/R$. R is an IR cutoff on the longitudinal momentum, and both R and N are taken to infinity to get eleven-dimensional physics. A state containing free massless particles indexed by i each with momenta (p_{-i}, \vec{p}_i) and spin (really, element within the supermultiplet) s_i has

$$P_+ = \sum_i \frac{(\vec{p}_i)^2}{2p_{-i}} \equiv R \sum_i \frac{(\vec{p}_i)^2}{2n_i}. \quad (2.4)$$

The corresponding wave function is approximately a product

$$\Psi = \prod_i e^{i\vec{p}_i \text{tr } \vec{X}^{(n_i)}/n_i} \psi_{n_i, s_i}(\vec{X}^{(n_i)}, \theta^{(n_i)}) \quad (2.5)$$

where $\vec{X} = \oplus_i \vec{X}^{(n_i)}$ is a direct sum over factors with rank n_i . It is strongly believed that there is a unique zero energy eigenstate $\psi_{n,s}$ for each $n \geq 1$ and s , so these wave functions are in one to one correspondance with asymptotic particle states.

A longitudinal five-brane is embedded in a four-dimensional hyperplane in the nine transverse dimensions. Let us take its coordinates to be X^m with $1 \leq m \leq 4$, and the transverse coordinates to be X^a with $5 \leq a \leq 9$. The manifest $SO(9)$ symmetry is broken to $SO(4)_{||} \times SO(5)_{\perp}$. The hyperplane will be $X^a = x_0^a$. Let ρ and $\dot{\rho}$ index the two spinor representations of $SO(4)_{||}$ and α index a spinor of $SO(5)_{\perp}$, all raised and lowered with antisymmetric ϵ symbols. A nine-dimensional spinor reduces to a ‘‘symplectically real’’ spinor satisfying $\eta^* = \epsilon \epsilon \eta$.

The five-brane breaks half of the original 32 supersymmetries, $\eta_{\alpha}^{\dot{\rho}}$ and $\tilde{\eta}_{\alpha}^{\dot{\rho}}$, and leaves unbroken η_{α}^{ρ} and $\tilde{\eta}_{\alpha}^{\rho}$. Thus it has a 2^8 -fold multiplet of ground states forming a representation of the broken supersymmetries. This will be represented by adding fermionic couplings θ_0 to the Lagrangian, which transform inhomogeneously under the broken supersymmetry $\tilde{\eta}_{\alpha}^{\dot{\rho}}$.

We propose to describe the five-brane by adding a complex boson $v^{\rho\dot{\rho}}$ and fermion $\chi^{\alpha\dot{\rho}}$, both vectors under $U(N)$ and symplectically real. The Lagrangian is

$$\begin{aligned} L_5 = & |D_t v^{\rho\dot{\rho}}|^2 + \chi D_t \chi - v_{\rho\dot{\rho}} (X^a - x_0^a)^2 v^{\rho\dot{\rho}} - \chi_{\alpha}^{\dot{\rho}} (X^a - x_0^a) \gamma_a^{\alpha\beta} \chi_{\beta\dot{\rho}} \\ & - v_{\rho\dot{\rho}} (\theta - \theta_0)_{\alpha}^{\rho} \chi^{\alpha\dot{\rho}} + v_{\rho\dot{\rho}} [X^m, X^n] \sigma_{mn}^{\rho\sigma} v_{\sigma}^{\dot{\rho}} - |v|^4. \end{aligned} \quad (2.6)$$

This is modeled after the dimensional reduction of $\mathcal{N} = 1$, $d = 6$ gauge theory and the last two terms here combine with the last term of (2.1) to form the usual D-terms of that theory. The mass term could be rotated to a conventional $d = 6$ mass term, but we choose to keep $SO(5)_{\perp}$ manifest.

The supersymmetries now act as (2.2) on X and

$$\begin{aligned} \delta v^{\rho\dot{\rho}} &= \eta_{\alpha}^{\rho} \chi^{\dot{\rho}\alpha} \\ \delta \chi^{\alpha\dot{\rho}} &= D_t v^{\rho\dot{\rho}} \eta_{\rho}^{\alpha} \\ \delta \theta_{\alpha}^{\rho} &= \delta_0 \theta_{\alpha}^{\rho} + v^{\dot{\rho}(\rho} v_{\dot{\rho}}^{\sigma)} \eta_{\sigma\alpha} \\ \delta \theta_{0\alpha}^{\rho} &= \tilde{\eta}_{\alpha}^{\rho} \\ \delta x_0^a &= -2\eta^{\rho\alpha} \gamma_{\alpha\beta}^a \theta_{0\rho}^{\beta} \\ \delta A &= \delta_0 A + 2\eta^{\rho\alpha} \theta_{0\rho\alpha} \end{aligned} \quad (2.7)$$

on the other fields. We have realized the algebra of the broken supersymmetry $\tilde{\eta}_\alpha^\rho$ and unbroken η_α^ρ manifestly on the parameters – alternatively, somewhat simpler transformation laws could be obtained by rewriting the Lagrangian in terms of combinations $X - x_0$, $\theta - \theta_0$ and $A - a_0$ invariant under the broken supersymmetry. The unbroken supersymmetry $\tilde{\eta}_\alpha^\rho$ does not act on the new sector. The broken supersymmetries η_α^ρ should also be realized in some non-linear way, but making this manifest appears to be more complicated.

By introducing auxiliary fields $D^{\rho\sigma}$, the Lagrangian can be made quadratic in v . This is an easy way to see that an overall coupling constant can be absorbed by field redefinition. The external fields of the five-brane will be produced by a one-loop effect, and thus there is no adjustable charge.

3. Dynamics of zero-branes

In this section we argue that the modifications to zero-brane dynamics produced by (2.6) agree with predictions from supergravity.

Let us first consider the ground state, a configuration in which all of the zero-branes are far from the five-brane, whose position we now take to be $x_0^a = 0$. The modes v and χ are all massive and sit in their ground states. By supersymmetry, their contributions to the vacuum energy cancel.

A graviton far from $x_0^a = 0$ can be studied using the effective Lagrangian produced by integrating out v and χ . We first check that this effective Lagrangian reproduces the metric of the five-brane [5,6] as felt by the bound state of N zero-branes. The light-cone zero-brane Lagrangian is

$$\mathcal{L}_0 = \frac{1}{2R}(D_t x_{||})^2 + \frac{1}{2R} \left(1 + \frac{B}{r^3}\right) (D_t x_{\perp})^2 \quad (3.1)$$

where the constant $B = \kappa_{11}^2 T_5 / 4\pi^2$, $T_5 = (\pi/2\kappa_{11}^4)^{1/3}$ in the conventions of [6], and κ_{11} is determined by the parameters of (2.1) in a way implicit in [1].

The computation of the one-loop effective Lagrangian for a single zero brane is the field theory limit of the 0–4-brane computation carried out in [4]. They reproduced the $d = 10$ dimensional reduction of this metric and showed that the long distance limit agreed with the known string theory result and four-brane tension. This is related to eleven-dimensional five-brane tension as $T_4 = RT_5$, while the gravitational couplings are related as $R/\kappa_{11}^2 = 1/\kappa_{10}^2$, so the result has the correct normalization. Since the new degrees of freedom are vectors of $U(N)$, the leading large distance, large N contribution to $\text{tr } \dot{X}^2$ will be independent of N , which combined with the scaling of [1] produces the

correct dependence on $p_{11} = N/R$. Thus a graviton feels the long distance metric of a five-brane.

As in [4], we would like to claim that this result is exact to all orders in perturbation theory. In the present case this is purely a question of the field theory defined by the combined Lagrangian (2.1) and (2.6). It is the dimensional reduction of $d = 4$, $\mathcal{N} = 2$ supersymmetric gauge theory for which this non-renormalization theorem is well-known, and we expect it to hold after dimensional reduction as well. We do not expect non-perturbative effects to change the behavior at distances much greater than the 11D Planck scale.

Local excitations of the five-brane world-volume will necessarily carry longitudinal momentum P^{11} in the IMF, and thus must be identified with threshold bound states of the zero-branes, localized around x_0^a . The full quantum Hilbert space of excitations will be reproduced by the same scheme of block diagonal wave functions as [1], allowing a new type of block for each five-brane and each number $N \geq 1$ of zero-branes.

Reproducing the five-brane spectrum requires a single tensor supermultiplet of bound states for each N , and there is some evidence for this conjectured spectrum. First, the same result is required for the eleven-dimensional interpretation of strongly coupled type IIA string theory. This leaves the question of whether the bound states are present in pure D-brane quantum mechanics. This was shown for $N = 1$ in [7,4], and the fact that the long range fields are the same as in the string theory makes the general statement very plausible.

This system is in some ways simpler than the pure 0-brane bound state dynamics considered in [1] and it would be quite interesting to formulate some of their physical conjectures here, especially those regarding Lorentz invariance.

4. The Dirac condition and Berry's phase

We now proceed to consider a membrane in this background. This will be a configuration with non-commuting expectation values for the longitudinal membrane coordinates, say X^5 and X^6 :

$$\begin{aligned} X^5 &= R_5 P & X^6 &= R_6 Q \\ [P, Q] &= 2\pi i. \end{aligned} \tag{4.1}$$

The other expectation values $X^1 \dots X^4$ and $X^7 \dots X^9$ are fixed to c-numbers x^1 , etc... to describe the membrane ground state.

The membrane should feel the five-brane magnetic field through the components $C_{56\mu}$ and the integrated world-volume coupling

$$\begin{aligned}\int C^{(3)} &= \int dt \partial_t X^\mu A_\mu(X) \\ A_\mu(X) &\equiv \int dX^5 dX^6 C_{56\mu}(X).\end{aligned}\tag{4.2}$$

Integrating out the vector degrees of freedom (2.6) must produce such a term in the effective action. The potential $A_\mu(X)$ will not be single-valued, so there must be an ambiguity in this procedure. This must be associated in some way with the point $X = 0$ where vector degrees of freedom become massless. Furthermore, if the membrane charge is correct, the magnetic flux $F = dA$ integrated over an S^2 surrounding the five-brane in the transverse dimensions (X^7, X^8, X^9) will be quantized in the minimal Dirac unit, $\int F = 2\pi$ [8].

To see these effects, we may consider motion $X(t)$ with extremely slow time dependence. The connection $A_\mu(X)$ is then Berry's connection [9],

$$A_\mu(X) = \langle X; 0 | \frac{\partial}{\partial X^\mu} | X; 0 \rangle\tag{4.3}$$

on the ground state wave function $|X; 0\rangle$.

Only the fermions see the direction of X^μ , so only they could produce the effect. Furthermore, the DNT argument can be made with a hyperplane only if it is infinite, and should be independent of any small fluctuations of the membrane. This suggests that the effect is due to fermion zero modes on the membrane.

In the limit of infinite membrane volume, we can realize the operators Q and P in the Schrödinger representation, $Q = \sigma$ and $P = 2\pi i \partial / \partial \sigma$, and turn the membrane theory into an effective two-dimensional field theory. The fermionic Hamiltonian derived from (2.6) becomes

$$H = \int d\sigma \bar{\chi}^{\dot{\rho}} \left(\gamma_5 2\pi i R_5 \partial / \partial \sigma + \gamma_6 R_6 \sigma + \vec{\gamma} \cdot \vec{X} \right) \chi_{\dot{\rho}},\tag{4.4}$$

where we use $\vec{X} = (X^7, X^8, X^9)$ to indicate the three dimensions transverse to both branes. At this point we drop the doublet index $\dot{\rho}$ and the symplectic reality condition, and work with a four component complex spinor. We then decompose this into the tensor product of two $SO(3)$ spinors, writing $\vec{\gamma} = -i\gamma_5\gamma_6\vec{\tau}$ in terms of Pauli matrices $\vec{\tau}$.

Now we can see the origin of the Berry phase corresponding to the five-brane magnetic field. Consider the effect due to a chiral (under $\gamma_5\gamma_6$) zero mode χ_0 of χ . We will show shortly that all other contributions to the Berry phase cancel for rigid motions of the membrane. It has a two-component wave function and the Hamiltonian

$$H = \bar{\chi}_0 \vec{X} \cdot \vec{\tau} \chi_0,\tag{4.5}$$

the same as for a spin 1/2 particle in the magnetic field $\vec{B} = \vec{X}$.

As is well known, the Berry connection (4.3) for this system is exactly the magnetic monopole with charge the Dirac quantum [9]. This is easy to verify by explicit computation of the ground state wave functions, as is done in textbooks [10]. The multi-valuedness of $A_\mu(X)$ arises because it is impossible to choose a single phase convention for the ground state everywhere on configuration space. This is a consequence of the degeneracy of the ground state at $X = 0$.

Thus, if we can show that membrane has a single chiral zero mode, it follows that the membrane and five-brane are both charged and satisfy the minimal Dirac condition. The non-zero mode contributions are obtained by working in a basis of eigenfunctions of $H_0 = \gamma_5 2\pi i R_5 \partial/\partial\sigma + \gamma_6 R_6 \sigma$. This is also very standard and the spectrum is $E_{n,\pm}^2 = n$ with the integer $n \geq 0$ for one chirality of $\gamma_5\gamma_6$ and $n \geq 1$ for the other chirality. Since the two chiralities produce opposite Berry phase, the non-zero mode contributions cancel, and only the single chiral mode $E_{0,+}$ contributes.

We should check that the zero-brane does not feel the Berry phase. Now the interaction $\chi\gamma_5\gamma_6\vec{X} \cdot \vec{\tau}\chi$ is completely symmetric under chirality reversal, so the phase completely cancels. More generally, the monopole field appears only with three transverse dimensions.

Similar fermionic zero modes will appear in a system containing a dual pair of a Dp -brane and a $D(6-p)$ -brane. Perhaps a variation of this argument can be found to show that they satisfy the minimal Dirac condition, as found in [11].

5. Conclusions

We proposed a modification of the M theory Lagrangian of [1] to describe a longitudinal five-brane, checked that a zero-brane sees the correct long-range fields, and checked that the membrane has the correct Dirac unit of charge. In combination these results also imply that the membrane tension is correct, a point recently verified by Shenker by a quite different argument [12]. Thus we have new evidence for the model's consistency.

A number of further tests can be done [13]. The local fields induced on the membrane world-volume can be computed. It should be possible to construct a membrane ending on the five-brane. It will also be interesting to interpret the Higgs branch of the zero-brane theory with multiple five-branes. The problem of constructing a ‘transverse’ five-brane (with x^{11} dependence) remains.

An unusual point of the construction is that our modified Lagrangian involves additional dynamical variables.* We believe that in generic situations they do not correspond

* This point was stressed to us by T. Banks, who also provided the resolution of the following paradox.

to additional physical degrees of freedom. An analogy can be drawn with the role of the off-diagonal matrix components in the original construction. In the D-brane context, states in which these variables are excited are states with physical stretched strings. In the M theory context, there should be no stretched string states before compactification. The resolution of this paradox is that these states have diverging energy in the eleven dimensional limit. This argument also applies to excitations of the vectors. It suggests that a modified Lagrangian without additional variables might exist in the limit.

Let us make some comments on the Dirac condition. A central theme of D-brane physics and its M theory analog is the equivalence between ‘bulk’ space-time interactions (exchange of closed strings between D-branes; the supergravity interaction in M theory) and quantum interactions of modes associated with pairs of branes (stretched open strings between D-branes, or off-diagonal matrix components and vectors in M theory).

Here we see this equivalence in its simplest form. The Dirac condition is one of the simplest and most general consequences of the combination of gauge theory and quantum mechanics – only the topological structure of the gauge field enters. On the other hand, Berry’s phase is one of the simplest aspects of the Born-Oppenheimer approximation in quantum mechanics, and is known to reflect topological structure of the configuration space. In particular, a singular connection can appear only if the adiabatic approximation breaks down, as it does at $\vec{X} = 0$. The simplest way this can happen is for two eigenvalues of the Hamiltonian to become degenerate, as in the present case. The $U(1)$ bundle defined by the phase of the wave function is embedded in a larger $SU(2)$ bundle at the origin, and there is a strong analogy with the realization of the Dirac monopole as the long distance field around the ’t Hooft-Polyakov monopole. This well-known analogy becomes a physical equivalence in our problem: the monopole field of the five-brane *is* the Berry’s phase monopole, and just as for the ’t Hooft-Polyakov monopole its singularity at the origin is regulated by embedding it in an $SU(2)$ bundle, but now this is just a larger subspace of the full quantum Hilbert space.

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